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Boson induced nuclear fusion in crystalline solids

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Abstract. In a calculation of demonstrative type collective, laser-like behavior of low energy nuclear fusion reaction of deuterons in crystalline environment is investigated. It is found that the reported extra ^{4}He production can be appropriately described with a model well known in quantum electronics in which the quantized boson (^{4}He) field interacts with an ensemble of two-level systems in a crystal resonator. The estimated life times of the two levels indicate that population inversion may be achieved. Thresholds of the deuteron number of the sample and of the electric current density of the pumping electrolysis are estimated in the calculation by analyzing the gain parameter and some other characteristics of the process. An explanation for the experimentally observed threshold behavior of the electric current density is given. A loss of a special type, that is the degenerate parametric amplifier mechanism, is suggested to be responsible for the difference between the expected and observed energies of the outgoing charged particles.

1 Introduction

This paper is motivated by the two decade old announcement [1], that excess heat due to nuclear fusion reaction of deuterons can be observed at deuterized Pd cathodes during electrolysis at near room temperature. The phenomenon of low energy nuclear fusion (LENF) reactions, summarized in references [2–5], is still doubted by most physicists due to the rather confused experimental situation. Recently, however, in a series of papers reproducible experimental evidence of LENF was reported [6].

In the first few years after the appearance of [1] a great number of efforts for the theoretical explanation of the effect were made (these are well summarized in [7]) but its full theoretical explanation is still missing (see the Scientific Overview of ICCF15 [8].) Therefore, recently, we have also tried to explain some of the basic questions of LENF reactions in solids [9] on the base of phonon exchange induced attraction [10] and solid state internal conversion processes [11].

The phonon exchange induced attraction between like charges is a well known phenomenon in solid state physics and it gives the ground to the theoretical explanation of superconductivity [12]. Adapting the phonon exchange induced electron-electron interaction potential for two quasi-free, heavy particles of like charges we obtained [9,10] their interaction potential $V_{ph}(\mathbf{K},\lambda)$ due to phonon exchange in the λth branch in $\mathbf{K}$ space as

$$V_{ph}\left(\mathbf{K},\lambda\right) = \left|g\left(\mathbf{K},\lambda\right)\right|^2 \hbar\omega_{\mathbf{q},\lambda}\left(D_1\left(\mathbf{k}_1\right) + D_2\left(\mathbf{k}_2\right)\right), \quad (1)$$

where $g(\mathbf{K},\lambda)$ is the particle-phonon coupling function, $\hbar\omega_{\mathbf{q},\lambda}$ is the energy of the exchanged phonon, $\mathbf{q}$ is the wave number vector in the first Brillouin zone ($\mathbf{K} = \mathbf{q}+\mathbf{G}$ outside the first Brillouin zone, where $\mathbf{G}$ is a vector of the reciprocal lattice), and

$$D_j\left(\mathbf{k}_j\right) = \frac{1}{\Delta E_j\left(\mathbf{k}_j,\mathbf{K}\right)^2 - \left(\hbar\omega_{\mathbf{q},\lambda}\right)^2}, \quad (2)$$

for $j = 1, 2$ with $\Delta E_j(\mathbf{k}_j,\mathbf{K}) = E_j(\mathbf{k}_j) - E_j(\mathbf{k}_j + \mathbf{K})$. Here $E_j = \hbar^2\mathbf{k}_j^2/(2M_j)$ is the kinetic energy and M_j is the rest mass of the quasi-free particle. (We can get back the phonon exchange induced electron-electron interaction potential substituting $M_j = m$, with m as the rest mass of the electron, into E_j.) As can be seen from (2) the interaction is attractive if $|\Delta E_j(\mathbf{k}_j,\mathbf{K})| < \hbar\omega_{\mathbf{q},\lambda}$. Heavy particles, e.g. protons, deuterons and other light nuclei, have much larger rest mass than the electronic rest mass ($M_j \gg m$) therefore this inequality is fulfilled in a much larger range of $\mathbf{K}$ than for electrons. Consequently the attraction is expected to be much stronger than in the case of electrons resulting a much deeper interaction potential (after Fourier transformation) in the real space.

According to our calculations the attractive potential, that arises due to phonon exchange between two quasi-free particles (that move in a crystalline material, such

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as e.g. Pd, which is partly filled in the interstitial lattice sites by deuterons), essentially compensates the effect of the Coulomb repulsion between them. It was found [9,10], that the attraction heavily depends on the parameter $u = n_d/n_{ion}$, the deuteron over metal ion number density. It was also recognized that the Gamow factor [13], that hinders nuclear reactions, e.g. nuclear fusion, between particles of like electric charge, strongly increases with increasing u and its hindering effect practically disappears at $u = 2$. (Our theory, according to the classification of [7], may be classified as a theory of barrier reduction due to lattice vibration.) As a conclusion, one can say that due to the phonon exchange generated attractive potential between fusionable particles their fusion reaction is possible at low, near room temperature. (Attraction due to phonons was already proposed by Schwinger in 1990 [14].)

It must be emphasized here that despite the strong attraction between the quasi-free particles (the depth of the potential can reach hundreds of eV) they can not form strongly bound pairs. Their binding energy E_{rel} in their relative motion must fulfill the $|E_{rel}| \ll \hbar\omega_O(u)$ condition, where $\hbar\omega_O(u)$ is the energy of the optical phonon (e.g. $\hbar\omega_O(u = 1) = 31$ meV [9,10].) Namely, if $|E_B| \to \hbar\omega_O(u)$ then the attraction disappears as can be seen from (2). Consequently the potential well formed by the sum of the attractive potential and the repulsive Coulomb potential (see e.g. Fig. 1 curve (b) in [9]) may cause some kind of weak pairing but not a strong binding. So one can say that the main consequence of the attraction due to phonon coupling is the sometimes drastic reduction of the width of the potential hill which they have to tunnel and the huge increase of the Gamow-factor. The proposed mechanism is fairly general and the calculations may be repeated for deuterized Ti, Ni crystals too and therefore it is reasonable to think that it can explain the possibility of low energy fusion reactions in general.

Motivated by the experimental observations that ordinary fusion products are missing in low energy fusion reactions we looked for a process which may partly be responsible for this fact. It is standard in nuclear physics that isomers of long life time mainly decay by internal conversion process in which an electron takes off the energy of the nuclear transition instead of a γ photon [15]. It was recognized that in a fusion of solid state internal conversion type [11], ordinary fusion products are missing, similarly to the normal internal conversion process, and charged particles help to get rid of the energy of the fusion reaction. In a solid, such as deuterized Pd, there are many possible particles that may take part in the solid state internal conversion process channel of the fusion reaction. The process was demonstrated through electron and deuteron assisted $p + d \to {}^3$He nuclear reactions. It was also shown that lattice effects can significantly increase the cross section of solid state internal conversion processes [16]. (Extra heat production can be partly attributed to the solid state internal conversion process too [9].)

In what was said above, fusion mechanisms, that are thought to be responsible for low energy fusion reactions, were sought for among individual processes of fusion. However, if the particles, which take part in the individual processes, are bosons, as e.g. it is the case of normal $d + d \to {}^4$He and charged particle (e.g. electron) assisted $d + d \to {}^4$He reactions, then their collective behavior may also be essential. In other words, their initial and/or final states must be described as part of a multi-particle (quantized) boson field. On the other hand, if there is a resonator present too, in our case it can be the crystal, then due to the (quantized) boson field, induced emission can take place and laser-like processes may be expected. Our theoretical attempt is strongly motivated by the observation that in the electric current density of the electrolysis threshold appeared [4,17], a fact that can indicate laser-like behavior.

We will focus our attention to the outgoing ^{4}He that has bosonic nature as its angular momentum/parity is 0^+. In order to demonstrate the idea, we investigate here the

$$e + d_{free} + d_{free} \to {}^4\text{He} + e' \quad (3)$$

nuclear fusion reaction, which is the $d + d \to {}^4$He reaction assisted in the solid state internal conversion process by an electron, in a crystal resonator. (The inverse reaction $e' + {}^4\text{He} \to e + d_{free} + d_{free}$ is also possible.) The processes, in which one party of the fusion process is a boson and induced emission plays essential role, we call boson induced fusion (BIF) reactions. There are many possible different BIF reactions, which can be distinguished by the participant particles, and accordingly reaction (3) is called electron assisted (^{4}He) BIF reaction.

In a demonstrative calculation it is shown that if LENF reactions of type (3) take place in a crystalline material then the behavior of the system is similar to that of a two-level atom ensemble coupled to the quantized field of a resonator. (Our model is similar in some aspect to the theory proposed in the early stage of cold fusion story [18] and it is closer to the one presented recently [19].) As was said above, the experimentally observed threshold in the electric current density of the electrolysis, that is considered to be the pumping mechanism, indicates laser-like behavior, therefore the condition of lasing is investigated and it is connected with the threshold of the gain of the boson field. Numerical results of the threshold number of the quasi-free (fusionable) deuterons, that must be present in the sample to start lasing, are determined and the threshold in the electric current density of the electrolysis is estimated. The problem of missing energetic charged fusion products is recognized and the degenerate parametric amplifier mechanism, a possible loss mechanism that may be responsible for this fact, is proposed. The importance of other BIF processes and their cross effects, furthermore the possible connection with an other cold fusion theory is also mentioned.

2 Electron assisted boson (^{4}He) induced fusion (BIF) reaction in crystal lattices

Bosons (^{4}He) are created in the vicinity of, or inside a crystal that, because of the Bragg law, may work like a resonator. The energy eigenstates of the boson in the resonator can be written as

$$\Psi_1(\mathbf{r}) = \frac{1}{H}\sqrt{\frac{2}{d_j}}\exp\left(i\mathbf{k}_{\parallel j}\cdot\mathbf{r}\right)\cos\left(\mathbf{k}_{\perp j}\cdot\mathbf{r}\right) \tag{4}$$

and

$$\Psi_2(\mathbf{r}) = \frac{1}{H}\sqrt{\frac{2}{d_j}}\exp\left(i\mathbf{k}_{\parallel j}\cdot\mathbf{r}\right)\sin\left(\mathbf{k}_{\perp j}\cdot\mathbf{r}\right). \tag{5}$$

Here $\mathbf{k}_{\parallel j}$ and $\mathbf{k}_{\perp j}$ are the components of the wave vector of ^{4}He parallel with and orthogonal to the jth crystal plane, $\mathbf{k} = \mathbf{k}_{\parallel j} + \mathbf{k}_{\perp j}$, $(\mathbf{k}_{\parallel j}\cdot\mathbf{k}_{\perp j} = 0)$. (The symbols $\parallel$ and $\perp$ refer to the components of any vector parallel with and orthogonal to the jth crystal plane.) With this notation the Bragg law has the form

$$2\left|\mathbf{k}_{\perp j}\right| = n_B\left|\mathbf{G}_j\right|, \tag{6}$$

where n_B is a natural number, $\mathbf{G}_j$ is a reciprocal lattice vector, and $|\mathbf{G}_j| = 2\pi/d_j$, where d_j is the distance of the resonator planes. We assume an open resonator with length $L = d_j$. The area of the resonator plates of linear dimension H is $H\times H$. The resonators select $\Psi_2(\mathbf{r})$, because the sine-like coupling has nodal points on the crystal planes thus absorption and outscattering are less intensive when compared it with the cosine-like case. (The phenomenon is well-known in the case of X-rays as anomalous X-ray transmission [20].)

The motion of the two deuterons and the electron, furthermore the motion of ^{4}He parallel with the resonator planes are treated quantum mechanically. A possible initial (upper) state of the two fusing deuterons and the initial electron with energy

$$E_a = 2m_{d0}c^2 + E_{kin,1} + E_{kin,2} + E_{e,i}, \tag{7}$$

is

$$|a\rangle = |d_1\rangle\otimes|d_2\rangle\otimes|e_i\rangle\otimes|0\rangle. \tag{8}$$

$m_{d0}c^2$ denotes the rest energy of the deuteron, $E_{kin,1}$, $E_{kin,2}$ are the initial kinetic energies of the two deuterons of states $|d_1\rangle$, $|d_2\rangle$, respectively, and $E_{e,i}$ and $|e_i\rangle$ are the energy and the initial state of the electron. The pumping mechanism, that prepares the ensemble of states $|a\rangle$ is electrolysis. A possible final (lower) state

$$|b\rangle = |0\rangle\otimes|0\rangle\otimes|e_f\rangle\otimes|\mathrm{He}_{\parallel}\rangle \tag{9}$$

with energy

$$E_b = m_{\mathrm{He}0}c^2 + E_{kin,\mathrm{He},\parallel} + E_{e,f}, \tag{10}$$

that is the sum of energy of motion of ^{4}He in directions parallel with the resonator planes and of the scattered electron. $E_{e,f}$ and $|e_f\rangle$ are the energy and state of the electron in the final state, respectively. $m_{\mathrm{He}0}c^2$ is the rest energy, $|\mathrm{He}_{\parallel}\rangle$ is the sate and

$$E_{kin,\mathrm{He},\parallel} = \frac{\hbar^2\mathbf{k}_{\parallel j}^2}{2m_{\mathrm{He}0}} \tag{11}$$

is the kinetic energy of the motion parallel with resonator planes of ^{4}He. The states $|b\rangle$ are degenerate states, as states $|e_f\rangle\otimes|\mathrm{He}_{\parallel}\rangle$ of different directions may have the same energy. (This degeneracy has to be taken into account later, in the cross section calculation by summing up over all the possible final states.) One pair of states $|a\rangle$ and $|b\rangle$ corresponds to a two-level system.

The part of the ^{4}He (bosonic) motion orthogonal to the resonator planes is field quantized, whose energy eigenvalue is

$$E_{kin,\mathrm{He},\perp}\left(\mathbf{k}_{\perp j}\right) = \frac{\hbar^2\mathbf{k}_{\perp j}^2}{2m_{\mathrm{He}0}}. \tag{12}$$

The corresponding Hamiltonian is oscillator type, $a^+(\mathbf{k}_{\perp j})$ and $a(\mathbf{k}_{\perp j})$ are the boson creation and annihilation operators in the mode with

$$\left[a\left(\mathbf{k}_{\perp i}\right), a^+\left(\mathbf{k}_{\perp j}\right)\right] = \delta_{ij}. \tag{13}$$

The orthogonal part of the state of ^{4}He can be described by number states $|n(\mathbf{k}_{\perp j})\rangle$.

The interaction of an ensemble of the above two-level systems with the quantized boson field of state $|n(\mathbf{k}_{\perp j})\rangle$ describing the perpendicular motion can be treated in a way known in quantum electronics [21]. The interaction Hamiltonian in the rotating wave approximation is

$$H_I = \sum_{\mathbf{k}_{\perp j},l}\left[g_l\left(\mathbf{k}_{\perp j}\right)a^+\left(\mathbf{k}_{\perp j}\right)\sigma_l + g_l^*\left(\mathbf{k}_{\perp j}\right)a\left(\mathbf{k}_{\perp j}\right)\sigma_l^+\right], \tag{14}$$

with

$$\sigma_l|a\rangle_l = |b\rangle_l, \tag{15}$$

and

$$\sigma_l^+|b\rangle_l = |a\rangle_l. \tag{16}$$

The interaction Hamiltonian couples the $\mathbf{k}_{\perp j}$th mode of the boson field and the lth two-level system, i.e. the direct product states $|A\rangle_l = |a\rangle_l\otimes|n(\mathbf{k}_{\perp j})\rangle$ and $|B\rangle_l = |b\rangle_l\otimes|n(\mathbf{k}_{\perp j})+1\rangle$. The coupling constant

$$g_l\left(\mathbf{k}_{\perp j}\right) = g_0\left(\mathbf{k}_{\perp j}\right)\exp\left(-G_W/2\right), \tag{17}$$

and $\exp(-G_W)$ is the Gamow-factor with

$$G_W = 2\pi\sqrt{\frac{\mu R_y}{m_e\left|U_0(0)\right|}}, \tag{18}$$

that has u dependence through $U_0(0)$, that is the depth of the optical phonon exchange induced attractive interaction potential between two quasi-free deuterons of the same kinetic energy $E_{kin,1} = E_{kin,2} = E_{kin}$ (see Fig. 2 of [9] and Fig. 5 of [10]). In (18) μ is the reduced mass of the two deuterons, m_e is the rest mass of the electron and R_y is the Rydberg energy. $g_0(\mathbf{k}_{\perp j})$ is determined by the matrix element V_{ab} of the Coulomb interaction governing solid state assisted nuclear fusion reaction as

$$V_{ab} = g_0\left(\mathbf{k}_{\perp j}\right)\exp\left(-G_W/2\right). \tag{19}$$

The Gamow-factor increases with increasing the localized deuteron concentration (u) in Pd.

In this model the parallel component of the ^{4}He motion is present in the lower level (see Eq. (9)). As a consequence there is a coupling between the parallel and perpendicular motions of ^{4}He. The transition $|A\rangle_l \to |B\rangle_l$ describes mainly nuclear fusion. It takes part in the build-up of the boson field. However, due to the coupling, in certain conditions, the induced emission through a laser-like process can increase the fusion rate.

The matrix element V_{ab} is calculated in a model similar to the one used in [11] and it is

$$V_{ab} = \int \varphi_i(\mathbf{r},\mathbf{R})\,\chi_i(\mathbf{r}_3)\,\varphi_f^*(\mathbf{r},\mathbf{R})\,\chi_f^*(\mathbf{r}_3) \times \left(V_{Cb}\left[\mathbf{R}-\frac{1}{2}\mathbf{r}-\mathbf{r}_3\right]+V_{Cb}\left[\mathbf{R}+\frac{1}{2}\mathbf{r}-\mathbf{r}_3\right]\right) d^3r d^3R d^3r_3. \quad (20)$$

Here V_{Cb} is the Coulomb potential, and the $\mathbf{r}_1$, $\mathbf{r}_2$ coordinates of the two deuterons are replaced by their relative $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$ and center of mass $\mathbf{R} = \mathbf{r}_1/2 + \mathbf{r}_2/2$ coordinates. With these coordinates the initial two deuteron state

$$\varphi_i(\mathbf{r},\mathbf{R}) = \frac{1}{\sqrt{V}} e^{i\mathbf{k}_i\cdot\mathbf{R}}\psi_{2d,i}(\mathbf{r}), \quad (21)$$

where $\psi_{2d,i}(\mathbf{r})$ is the initial state in the relative coordinates of the two deuterons that has the form

$$\psi_{2d,i} = (4\pi)^{-1/2} r^{-1} u_n(r_h)\, e^{-G_W/2} \quad (22)$$

in the $r \le R_0$ region, with R_0 as the radius of ^{4}He and $r = |\mathbf{r}|$. The classical turning point r_h is determined by the condition

$$\frac{e^2}{r_h} + U_0(r_h) = 0 \quad (23)$$

(the lower root in Fig. 6 in [10]). $u_n(r)$ is the solution of the reduced Schrödinger equation

$$-\frac{\hbar^2}{2\mu}\frac{d^2u_n(r)}{dr^2} + \left[\frac{e^2}{r}+U_0(r)\right]u_n(r) = E_{rel}u_n(r) \quad (24)$$

with energy eigenvalue $|E_{rel}| < \Delta E/2$ (for ΔE see later) valid for $r > r_h$. An approximate solution of $u_n(r)$ of the form

$$u_n(r) = \sqrt{\beta/\left(2^n n!\sqrt{\pi}\right)}\,e^{-\beta^2(r-r_0)^2/2}H_n\left[\beta(r-r_0)\right] \quad (25)$$

can be obtained [22] approximating $e^2/r+U_0(r)$ (see Fig. 6 in [10]) with an oscillator potential, whose minimum is fitted to the minimum of $e^2/r + U_0(r)$ at $r = r_0$. H_n is an Hermite polynomial and $\beta = \sqrt{\mu\omega/\hbar}$ with ω as the corresponding angular frequency of the oscillator. The initial electronic state is

$$\chi_i(\mathbf{r}_3) = \frac{1}{\sqrt{V}}e^{i\mathbf{k}_e\cdot\mathbf{r}_3} \quad (26)$$

where V is the volume of normalization, $\mathbf{k}_e$ and $\mathbf{r}_3$ are the wave vector and coordinate of the electron.

The final state of ^{4}He is

$$\varphi_f(\mathbf{r},\mathbf{R}) = \varPhi_{\text{He}}(\mathbf{r})\,\frac{1}{H}e^{i\mathbf{k}_{\parallel j}\cdot\mathbf{R}}\sqrt{\frac{2}{d_j}}\sin\left(\mathbf{k}_{\perp j}\cdot\mathbf{R}\right), \quad (27)$$

where $\varPhi_{\text{He}}(\mathbf{r})$ is the $\mathbf{r}$ dependent part of the ^{4}He wave function. For the function $\varPhi_{\text{He}}$ we apply the Weisskopf approximation, i.e. $\varPhi_{\text{He}} = \sqrt{3/(4\pi R_0^3)}$ if $r \le R_0$ and $\varPhi_{\text{He}} = 0$ if $r > R_0$. The final electronic state is

$$\chi_f(\mathbf{r}_3) = \frac{1}{\sqrt{V}}e^{i\mathbf{k}_3\cdot\mathbf{r}_3} \quad (28)$$

where $\mathbf{k}_3$ is the wave vector of the outgoing electron. Using that $|\mathbf{k}_i| \approx |\mathbf{k}_e| \ll |\mathbf{k}_3|$ and the dipole approximation in the integral over the relative nuclear coordinates we obtain

$$V_{ab} = \frac{64\pi^3e^2\sqrt{3R_0}\,\delta_2\left(\mathbf{k}_{\parallel j}+\mathbf{k}_{3\parallel}\right)}{V^{2/3}H\sqrt{L}\left|\mathbf{k}_{3\parallel}\right|^2\left|\mathbf{k}_{\perp j}\right|}u_n(r_h)\,e^{-G_W/2}, \quad (29)$$

where $\delta_2(\mathbf{k}_{\parallel j}+\mathbf{k}_{3\parallel})$ is a two dimensional Dirac-delta, and e is the elementary charge. Thus we obtain

$$g_0(\mathbf{k}_{\perp j}) = \frac{64\pi^3e^2\sqrt{3R_0}\,\delta_2\left(\mathbf{k}_{\parallel j}+\mathbf{k}_{3\parallel}\right)}{V^{2/3}H\sqrt{L}\left|\mathbf{k}_{3\parallel}\right|^2\left|\mathbf{k}_{\perp j}\right|}u_n(r_h). \quad (30)$$

Consequently in the state after fusion $\mathbf{k}_{3\parallel} = -\,\mathbf{k}_{\parallel j}$.

The energy relations are as follows. The nuclear reaction energy $Q = 2m_{d0}c^2 - m_{\text{He}0}c^2 = 23.846$ MeV, and it is shared between the ^{4}He and the electron as 75.7 keV and 23.77 MeV, respectively, in a normal single reaction. ($E_{kin,1}$ and $E_{kin,2}$ are much less than Q). E_B is the energy of a bound deuteron state at the octahedral sites in the Pd lattice of face-centered cubic (fcc) structure. The barrier height E_{barr} between the minima is 0.23 eV [23]. Thus $E_B = -E_{barr} + 3\hbar\omega_d/2$ relative to the top of the barrier, where $\hbar\omega_d = 48$ meV [11] producing the energy of a ground state of an oscillator $3\hbar\omega_d/2 = 72$ meV [24]. The number density of deuterons in this state is n_d. The bound deuterons are responsible for the optical phonon branch, they cause the attractive optical phonon exchange induced potential. Its depth $U_0(0)$ as a function of E_{kin} has a sharp minimum at $E_{kin} = \hbar\omega_O(u)$ [9,10], which is the energy of the optical phonon (e.g. $\hbar\omega_O(u = 1) = 31$ meV), $E_{kin} = E_{kin,1} = E_{kin,2}$ is the energy of the quasi-free particles measured from the top of the barrier. The minimum in $U_0(0)$ at $E_{kin} = \hbar\omega_O(u)$ causes a peak in the Gamow-factor $\exp(-G_W)$ (see Eq. (18)) the width of which determines the energy width ΔE of the "lasing" of the ensemble of two-level systems. The numerical calculation for ΔE results a few meV around $u = 1$.

Our aim is to estimate the threshold current of the electrolysis, that is the pumping mechanism of BIF, therefore instead of a rigorous quantum optical laser calculation [21] we do it in the simplest manner. Therefore we investigate the gain that determines the threshold number density $n_{a,th}$ of the number density n_a of the states $|a\rangle_l$ by the threshold condition of the gain [25].

3 Threshold condition of electron assisted (^{4}He) BIF

First the conditions of BIF (the conditions of "lasing") must be investigated. The life times of states $|a\rangle$ and $|b\rangle$ are τ_a and τ_b, respectively. τ_a is mainly determined by the characteristic time $\tau_{ph} \simeq 100$ fs [26] of phonon electron interaction since the characteristic time τ_{ab} of the spontaneous transition $|a\rangle \rightarrow |b\rangle$ is so large that $\tau_{ab} \gg \tau_{a,ph} \sim \tau_{ph}$ for every u. Therefore one can take $\tau_a = \tau_{ph}$ resulting $dE_a = 2\pi\hbar/\tau_a = 1.05$ meV. τ_b is the characteristic time of energy loss of the outgoing electron determined by the

$$\Delta E = \left(\frac{dE}{dx}\right)_{loss,e} v_e \tau_b \tag{31}$$

condition, where v_e is the velocity and $(dE/dx)_{loss,e}$ is the energy loss per length of the electron with $v_e \simeq c$, the velocity of light. For the energy E_e of the outgoing electron $E_e \simeq Q$. $(dE/dx)_{loss,e}$ is mainly the result of bremsstrahlung [27,28]. In the following $\Delta E = 1.0$ meV is used. It is in accordance with the fact that the magnitude of the effective energy interval for the phonon exchange between the initial free deuterons is a few meV as it was obtained investigating the u dependence of the Gamow-factor. Carrying out the calculation for Pd of charge number $Z = 46$ and volume of unit cell $v_c = d_{Pd}^3/4$ with $d_{Pd} = 3.89 \times 10^{-8}$ cm, that is the lattice parameter, we obtain $\tau_b \simeq 4 \times 10^{-18}$ s. As a result, $\tau_a/\tau_b \gtrsim 2.5 \times 10^4$ that fulfills the condition of population inversion.

In our case the steady state gain [25]

$$G = \sigma_{ab} n_a - \alpha_{loss}, \tag{32}$$

where σ_{ab} is the cross section of the induced transition $a \rightarrow b$, n_a is the steady state number density of systems in state $|a\rangle$ and α_{loss} is the loss parameter. The threshold value G_{th} of the gain, at which the BIF process begins, is determined by the

$$G_{th} = -\ln r_c/L \tag{33}$$

condition [25], where r_c is the reflection coefficient of the planes. Standing waves correspond to $r_c = 0.5$ and $t_c = 0.5$ at each plane (t_c is the transmission coefficient) and consequently if we take $L = d_r$ than neighboring resonators work like one resonator with $r_c = 1$, therefore $G_{th} = 0$ is the condition of threshold of BIF process, that gives

$$n_{a,th}\sigma_{ab} = \alpha_{loss} \tag{34}$$

for the threshold number density $n_{a,th}$ of the number density n_a.

For the energy loss per length $\varepsilon = 12$ eV/Å was obtained for He$^+$ ions of energy 25 keV channeling in Au foil [29]. In the experiment both planar and directional channeling were present. We estimate α_{loss} taking

$$\alpha_{loss} = \frac{\varepsilon}{\Delta E}\frac{E_{kin,\mathrm{He},\perp}}{E_{kin,\mathrm{He},\parallel}}. \tag{35}$$

As the charge number of Au is larger than the charge number of Pd so the energy loss is overestimated in that manner. It is expected, that the possible lowest value of n_B in the Bragg law is effective because in this case the relative (referred to the lattice plane distance) displacements of Pd atoms caused by thermal motion are less disturbing, furthermore the loss is the smallest as can be seen from equation (35), therefore the smallest $E_{kin,\mathrm{He},\perp}$ corresponding to $n_B = 1$ with $d_j = d_{Pd}$ is used in the numerical calculation.

The transition probability per unit time W_{ab} of the process can be obtained from (29) with standard methods with the modification that for the sums over the possible final states in determining W_{ab} the

$$\sum_{\mathbf{k}_{\parallel,\mathrm{He}}} \rightarrow [H/(2\pi)]^2 \int d^2\mathbf{k}_{\parallel,\mathrm{He}} \tag{36}$$

and

$$\sum_{\mathbf{k}_{\parallel,e}} \rightarrow \int [H/(2\pi)]^2 \int d^2\mathbf{k}_{\parallel,e} \tag{37}$$

substitutions are applied, where $H \times H$ is the area of the resonator plates of linear dimension H. The cross section σ_{ab} is determined from W_{ab} as usual, i.e. $\sigma_{ab} = W_{ab}/F_{\mathrm{He}}$, where $F_{\mathrm{He}} = v_\perp/(H^2 L)$ is the corresponding ^{4}He flux represented by the motion of one particle of velocity $v_\perp$ in the $\perp$ direction.

The number N_a of the systems in the upper state is

$$N_a = N_{e,i} N_{d,las} \left(N_{d,las} - 1\right)/2 \simeq N_{e,i} N_{d,las}^2/2, \tag{38}$$

where $N_{d,las}$ is the number of quasi-free deuterons, that can take place in the BIF process, and $N_{e,i}$ is the number of initial electrons in the sample, that may be taken into account in the initial state. $N_{d,las}$ is approximately determined as $N_{d,las} = N_d \xi$, where N_d is the steady state number of quasi-free deuterons in the sample, and the notation $\xi = \int_{\Delta E} f_d(E_{kin})\, dE_{kin}$ is introduced, where $f_d(E_{kin})$ denotes the energy distribution function of quasi-free deuterons. Using Bose-Einstein distribution of a free deuteron gas we obtain $\xi(u = 1) = 0.0078$ at room temperature ($k_{\mathrm{B}}T = 25$ meV). Thus the threshold number $N_{a,th}$ of N_a is $N_{a,th} = N_{e,i} N_{d,th}^2 \xi^2 \delta/2$, where $N_{d,th}$ is the threshold steady state number of quasi-free deuterons in the sample, and $\delta = \delta\Omega/(4\pi)$ is a solid angle correction factor, which takes into account the requirement that the wave vectors of the initial deuterons must be approximately parallel [10], where $\delta\Omega$ is the allowed solid angle deviation. We have used $\delta = 2.5 \times 10^{-3}$. Thus $n_{a,th} = n_{e,i} N_{d,th}^2 \xi^2 \delta/2$, where $n_{e,i} = 10/v_c$ is the number density of the initial ten $4d$ electrons of Pd. Substituting everything into equation (34) one obtains

$$N_{d,th} = K \frac{V_s^{1/3}}{\xi u_n\left(r_h\right) e^{-G_W/2}\sqrt{\delta}} \tag{39}$$

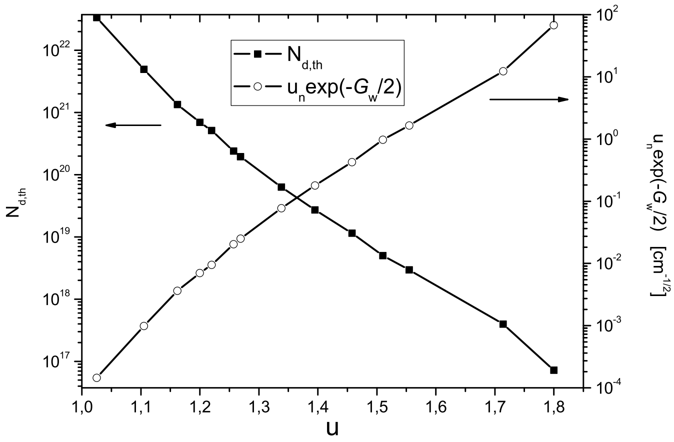

Fig. 1. The u dependence of the threshold deuteron number $N_{d,th}$ of the sample and of $u_n(r_h)e^{-G_W/2}$ [cm$^{-1/2}$] (see the text) of BIF. $u = n_d/n_{ion}$ is the deuteron over metal ion number density.

for the threshold deuteron number of BIF in the sample, where $K = 7.6 \times 10^{14}$ cm$^{-3/2}$ and V_s is the volume of the sample. $N_{d,th}$ can be connected with the threshold electric current density j_{th} of the electrolysis as $j_{th} = eN_{d,th}/(F\tau_{rec})$, where τ_{rec} is the time of recombination of a quasi-free deuteron into a bound state, F is the surface of the Pd cathode and e is the elementary charge. As a typical value we take $V_s^{1/3}F^{-1} = 0.15$ cm^{-1} that corresponds to a Pd rod of length 5 cm and of diameter 0.3 cm [17]. Thus j_{th}[A/cm^2] $= 0.165 \times u_n^{-1}(r_h)e^{G_W/2}$[cm$^{1/2}$]$/\tau_{rec}$[s]. τ_{rec} can be obtained from the deuteron mobility $\mu_d = e\tau_{rec}/m_{d0}$ and the diffusion constant D through the $\mu_d k_B T = eD$ Einstein-expression [30] as $\tau_{rec} = m_{d0}D/(k_B T)$. Substituting $D = 6 \times 10^7$ cm^2/s [31] obtained at $u < 1$, we get $\tau_{rec} \sim 5 \times 10^{-3}$ s.

In Figure 1 the u dependence of $u_n(r_h)e^{-G_W/2}$ [cm$^{-1/2}$] and $N_{d,th}$ are given. The obtained $N_{d,th}$ values lie in a realistic range. With the highest value of $u_n(r_h)e^{-G_W/2} = 67.7$ cm$^{-1/2}$ obtained at $u = 1.8$ we get the observed $j_{th} = 0.35$ A/cm^2 value [4,17] with $\tau_{rec} = 7.0 \times 10^{-3}$ s. It is reasonable to suppose that above $u = 1$, which is the concentration of filled octahedral interstitial sites, τ_{rec} increases significantly, as approaching $u = 1$ the empty interstitial lattice sites decrease and so a quasi-free deuteron needs more time in order to be able to find a vacant site. Therefore it is expected that the observed threshold current density can be reached at much lower value of u.

4 Discussion

The fit of the results of numerical calculations of the threshold current density with the observations indicates a good coincidence. This is the main goal of this work since it indicates that mechanisms of BIF type may have realistic ground.

In order to have an outlook for the efficiency of BIF we enumerate from the family of the possible BIF reactions two that may also be responsible for ^{4}He and/or heat production. One such reaction is the deuteron assisted BIF

$$d_{bound} + d_{free} + d_{free} \rightarrow \ ^4\text{He} + d' \quad (40)$$

in which a localized (bound) deuteron sitting initially in an interstitial lattice site assists the BIF process. This process has a further advantage as the outgoing deuteron is also a boson. So it may also interact with the crystal resonator and the part of the outgoing deuteron (also bosonic) motion orthogonal to the resonator planes may be also field quantized. In this process the induced emission both in the quantized ^{4}He and in the quantized deuteron modes may work and two coupled modes are built up in the resonator. (Process of that type may be called double BIF.) These processes may be treated in a more complicated two mode laser model different from the one discussed here for the sake of demonstration of the main thought of BIF. (In process (40) the 23.846 MeV reaction energy is shared between the ^{4}He and the deuteron in a ratio 1:2, the ^{4}He and the deuteron will have 7.949 MeV and 15.897 MeV energy initially after fusion, respectively.)

From the point of view of total heat production the deuteron assisted $p + d \rightarrow \ ^3$He reaction

$$d_{bound} + p_{free} + d_{free} \rightarrow \ ^3\text{He} + d' \quad (41)$$

must be also partly taken into account. In this process an initially bound deuteron assists the fusion reaction too but now BIF may work only in the outgoing deuteron mode. In [9] it was obtained that the Gamow factor of the $p + d \rightarrow \ ^3$He process increases most rapidly and therefore the deuteron assisted BIF may be partly responsible for heat production. Furthermore, if the double BIF with outgoing deuteron works than its quantized deuteron mode, as a cross effect, may couple to and induce reaction (41).

The fast electrons having 23.77 MeV energy, that are expected in (3), were not found in observations. One of the possible explanation is the degenerate parametric amplifier mechanism [32] that may couple the quantized perpendicular to resonator plates motion with the motion of charged particles parallel with resonator plates causing their deceleration. It is expected that the degenerate parametric amplifier mechanism is mainly responsible for loss in BIF processes in general and it can explain the essentially lower energies of the outgoing charged particles that are expected in (40) and (41) processes too. (This mechanism may be connected with the loss term $\Gamma(E)$ introduced by [19] in their basic model showing promising results in the numerical evaluation.)

In order to determine the relationship between their rates, their threshold currents and their cross effects a detailed and systematic analysis of all the possible and competing BIF processes is necessary. The coupling of the different BIF processes may lead to strong nonlinearities and phenomena similar to the processes that appear in nonlinear and quantum optics because of the coupling of two or more laser modes (see e.g. [33]). Considering that the reaction, which will be actually started corresponds to the process of lowest threshold current, it is most important to determine the relationship between the threshold currents. (The situation is similar to a laser-material, that is pumped uniformly and which has more than one lasing transition. The laser will work in the mode which has the smallest threshold number density of the corresponding upper state.)

Nevertheless we can conclude from the surprisingly good qualitative agreement between the observed threshold current and the calculated one obtained in the demonstrative model discussed here that the importance of BIF reactions in explaining ^{4}He and heat production in LENF processes can not be doubted. The family of BIF processes may explain some disturbing characteristics of LENF observations. First, one might expect that if low energy nuclear fusion reactions are observed, i.e. if the particles overcome Coulomb repulsion, than all the possible fusion reactions must be observed with a rate proportional to their individual cross sections. However, ^{4}He production was not accompanied with the expected rate of other fusion products. But as a consequence of induced emission in the boson field the rate of any BIF process may come out of the rate of the normal nuclear fusion processes and may explain e.g. the lack of the expected rate of neutrons. Also, BIF may qualitatively explain the die away of the effect with increasing time of electrolysis as the large energy density dissipated by the emerging energetic fusion products may damage the crystal, create crystal defects, e.g. dislocations, the increasing density of which decreases the number of possible resonators leading to a halt in BIF. Furthermore, the mechanism heavily depends on u through the u dependence of the threshold current density that, however, may have other, unknown dependences, e.g. dependence on the density and type of lattice defects, dependence on the grain size, etc. Therefore the detailed analysis of BIF and the loss mechanism proposed may help to give new guide-lines to plan more informative and reproducible experiments and possibly may help in the deeper and better understanding of LENF reactions.

5 Summary

In order to demonstrate the idea of BIF the analogy between an electron assisted BIF reaction, that may take place in deuterized metal environment during electrolysis, and the system of quantized photon field coupled with a two-level atom ensemble in a resonator was discussed. The possibility of population inversion was pointed out. Adapting the gain condition of quantum electronics and calculating the cross section of the boson induced process the u dependence of the threshold of the quasi-free deuteron number $N_{d,th}$ of the sample, that is necessary to the laser like BIF process, was determined. With the aid of the $N_{d,th}$ numbers the threshold of the electric current density of the electrolysis was estimated and compared with the experimentally observed value. On the basis of the obtained realistic values of $N_{d,th}$ and of the good agreement between the estimated and the experimentally observed threshold electric currents one can say that when fusion reactions happen in deuterized metal environment of crystalline structure during electrolysis, the effect of boson induced nuclear fusion processes may be responsible for the observed extra ^{4}He and heat production. A degenerate parametric amplifier loss mechanism is put forward in order to explain the lack of expected fast electrons and the difference between the expected and observed energies of the outgoing charged fusion products. Other possible BIF reactions and their cross mechanisms are also discussed.

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